Position measurements and response uniformity of the LAr FCal prototype for ATLAS


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Radical move - replacing of the parallel gap of the ionization detector with the tubular one.
Prototype module with tubes inserted

Prototype-95: fully instrumented
Module equipped with readout board, cables and excluder
CERN-93 and CERN-95

Changes:
- Full length EM module
- High performance MWPC telescope
- Wide beam
- Hadronic Tail Catcher instead of 'random' counters
MWPC telescope precision studies

The observed deviation of the hit (cluster of wires) in the chamber may be presented as a (quadratic) sum of the chamber precision and precision of the track reconstruction by the rest of the chambers. So we have a set of 20 equations for X and 20 equations for Y coordinate (i=1,2,3,4,5 and \( n^{\text{wires}} = 1,2,3,4 \)):

\[
R_{m}^{2}(i,n^{\text{wires}}) = R_{i}^{2}(i,n^{\text{wires}}) + R_{f_i}^{2}(z_{j}, R_{j}^{2}(j,n^{\text{wires}}), j \neq i)
\]

Solution of those equations is presented in table below

<table>
<thead>
<tr>
<th>Cluster →</th>
<th>1 wire</th>
<th>2 wires</th>
<th>3 wires</th>
<th>4 wires</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane</td>
<td>RMS (mm)</td>
<td>events</td>
<td>RMS (mm)</td>
<td>events</td>
</tr>
<tr>
<td>X_1</td>
<td>12.5</td>
<td>0.32</td>
<td>94.5%</td>
<td>0.29</td>
</tr>
<tr>
<td>X_2</td>
<td>9.2</td>
<td>0.33</td>
<td>77.5%</td>
<td>0.27</td>
</tr>
<tr>
<td>X_3</td>
<td>6.0</td>
<td>0.38</td>
<td>72.1%</td>
<td>0.31</td>
</tr>
<tr>
<td>X_4</td>
<td>2.6</td>
<td>0.37</td>
<td>78.7%</td>
<td>0.42</td>
</tr>
<tr>
<td>X_5</td>
<td>2.5</td>
<td>0.32</td>
<td>65.3%</td>
<td>0.41</td>
</tr>
<tr>
<td>X-Precision</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y_1</td>
<td>12.5</td>
<td>0.32</td>
<td>83.8%</td>
<td>0.29</td>
</tr>
<tr>
<td>Y_2</td>
<td>9.2</td>
<td>0.29</td>
<td>93.9%</td>
<td>0.25</td>
</tr>
<tr>
<td>Y_3</td>
<td>6.0</td>
<td>0.38</td>
<td>78.9%</td>
<td>0.48</td>
</tr>
<tr>
<td>Y_4</td>
<td>2.6</td>
<td>0.34</td>
<td>74.7%</td>
<td>0.46</td>
</tr>
<tr>
<td>Y_5</td>
<td>2.5</td>
<td>0.32</td>
<td>77.8%</td>
<td>0.39</td>
</tr>
<tr>
<td>Y-Precision</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Event from the ‘residual’ tail - compact low amplitude cluster with some sprays around.

Another example of scattered beam electron, accompanied by photon (single track event with two clean clusters in the FCal
High angular (thanks to long base) resolution of the MWPC telescope allows to apply

The Beam Envelope Criteria

\[ R_{\text{env}} = (\Delta \theta_x / \Delta_x)^2 + (\Delta \theta_y / \Delta_y)^2 < 1. \]

The idea is based on the fact that impact angle \((\theta_x, \theta_y)\) depends on the impact point - due to geometry of the wide defocused beam that was used.
The correlation may be presented by simple equations, describing the ‘cones’ of the (de)focused beam:

\[ \theta^0_X = \frac{\delta \theta_X}{\delta X} \left( X_{IMPACT} - X_0 \right) \]

\[ \theta^0_Y = \frac{\delta \theta_Y}{\delta Y} \left( Y_{IMPACT} - Y_0 \right) \]

\[ R_{ENV} = \left( \frac{\theta_X - \theta^0_X}{\Delta X} \right)^2 + \left( \frac{\theta_Y - \theta^0_Y}{\Delta Y} \right)^2 \]

The Envelope Parameter \( R_{env} \) shows how far the given track is ‘away from it’s expected direction;.’
Off-line particles identification allows to determine the hadronic (pions) contamination of the electron data.

Fake 'electrons' from the pure hadronic beam were used
Position reconstruction precision and Beam Envelope track selection. 193 GeV electrons, -0.6° (left) and -3.6° (right) beam impact.
Traditionally used methods of the Shower position reconstruction

\[ X_{\text{pred}}^{\text{non-lin}} = \frac{\sum x_i \times f(A_i)}{\sum f(A_i)} \]
\[ Y_{\text{pred}}^{\text{non-lin}} = \frac{\sum y_i \times f(A_i)}{\sum f(A_i)} \]

Non-linear weighting of amplitudes (logarithm is commonly used) works well on typical S-shapes.

\[ X_{\text{COG}} = \frac{\sum x_i \times A_i}{\sum A_i} \]
\[ Y_{\text{COG}} = \frac{\sum y_i \times A_i}{\sum A_i} \]

Basic Center of Gravity (COG)

COG with 1-d angle-depending correction - good for regular structures like crystal based calorimeters without large local non-uniformity.

\[ X_{\text{corr}} = X_{\text{COG}} + \Delta_x(X_{\text{COG}}, \theta) \]
\[ Y_{\text{corr}} = Y_{\text{COG}} + \Delta_y(Y_{\text{COG}}, \theta) \]

COG with 2-d angle depending correction.
The Center of Gravity method shows very complex 2-d pattern of systematic errors...

Systematic errors of the CoG method. 193GeV electrons, $-0.6^\circ$ beam impact angle. The non-zero impact angle causes the asymmetry of the shower projection to the calorimeter 'face'—and so comes the asymmetry of the 'S-shape'.
More advanced method was used for shower position reconstruction in the Prototype...

For each pair of channels parameter $F_{ij}$ may be used to determine area where impact could take place:

$$ F_{ij} = \frac{(A_i - A_j)}{(A_i + A_j)} $$

Area where the impact point $(X_{HIT}, Y_{HIT})$ may be located relatively to the chosen pair of channels (i,j) $(X_{ij}, Y_{ij})$ is described by set of function $P_{ij}$ (partial likelihood functions):

$$ P_{ij}(X_{HIT} - X_{ij}, Y_{HIT} - X_{ij}, F_{ij}) $$

used to reconstruct the complete likelihood function:

$$ P(X_{HIT}, Y_{HIT}) = \prod_{i,j=1}^{N} P_{ij}(X_{HIT} - X_{ij}, Y_{HIT} - X_{ij}, F_{ij}) $$

For the $P(X_{HIT}, Y_{HIT})$, extracting the peak position, center of gravity or the fit center parameter to get the estimation of the impact point position.
Example of 2-dimensional
$P_{ij}\left(X_{HIT} - X_{ij}, Y_{HIT} - X_{ij}, F_{ij}\right)$ set (CERN-95, 100 GeV electrons, $0.4^\circ$ impact angle).
Example of position reconstruction sequence - Steps 1, 2

Superposition of two Partial Likelihood Functions in the event.

Zoomed area from the upper plot
Example of position reconstruction sequence - Steps 7,8
Example of position reconstruction sequence - Steps 11, 12
Position resolution as a function of electron energy.

No (extra) dead material in front of the cryostat

\[ \Delta x = \Delta_0 \bigoplus \frac{\Delta_S}{\sqrt{E}} \bigoplus \frac{\Delta_N}{E} \]
Comparison of three methods of position reconstruction used in the CERN-95 data analysis

- Center of Gravity
- Corrected Center of Gravity
- Likelihood

Position resolution $\Delta Y$, RMS (mm)

Tracker $Y$-resolution 0.375 mm

$E^{-1/2}$ (GeV)
Summary of position resolution parametrization

\[ \Delta x = \Delta_0 \oplus \frac{\Delta_S}{\sqrt{E}} \oplus \frac{\Delta_N}{E} \]

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>(\Delta x)</th>
<th>(\Delta y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Delta_0) (mm)</td>
<td>(\Delta_S) (mm)</td>
</tr>
<tr>
<td>3.6°</td>
<td>1.166±0.002</td>
<td>0.0±5.0</td>
</tr>
<tr>
<td>2.4°</td>
<td>0.874±0.009</td>
<td>3.63±0.21</td>
</tr>
<tr>
<td>1.6°</td>
<td>0.705±0.006</td>
<td>4.75±0.11</td>
</tr>
<tr>
<td>0.6°</td>
<td>0.523±0.005</td>
<td>4.81±0.074</td>
</tr>
<tr>
<td>0.4°</td>
<td>0.402±0.010</td>
<td>5.14±0.040</td>
</tr>
</tbody>
</table>
The Prototype-95 electron position resolution as a function of angle and direct $X_0$ measurements.

The 'radial' resolution ($X$) has a term $X_0 \sin(\theta)$...

... while the azimuthal ($Y$) does not indicate angular variations.
Position reconstruction for 200 GeV pions, beam impact angle \(-3.6^0\)

(Events with less than 20 GeV in the EM section were discarded)

\[ \Delta Y \text{ RMS} = 2.18 \text{ mm} \]

\[ \Delta X \text{ RMS} = 4.94 \text{ mm} \]
Position resolution for 200 GeV hadrons as a function of angle

\[ \Delta X = 1.808 \times 72.457 \times \Theta \ (\text{mm}) \]

\[ \Delta Y = 1.878 \times 4.409 \times \Theta \ (\text{mm}) \]
Position resolution ($\Delta X$ and $\Delta Y$ RMS) as a function of dead material, located in front of the Prototype module.

193 GeV electrons, -3.6° impact angle. CERN-95 data.
The relative response variations within a single readout channel.

Presented maps correspond to -0.6° (upper plot) and -3.6° (lower one) impact angle.
The single channel response map (left plots) and single tube response map (right plots)

Upper plots correspond to impact angle -0.6°, lower ones - to -3.6°.
X and Y slices of reconstructed single tube response

X- and Y-slices for impact angle \(-0.6^\circ\).
Why not consider the Tube as a sum of identical tiny sensitive ‘straws’ of liquid argon?

Same way as channel is a sum of four identical tubes...

$$A_\text{tube}^f(x, y) = \int_{\phi=0}^{\phi=2\pi} A^f(x - R \cos \phi, y - R \sin \phi) d\phi$$

$$A_\text{tube}^f(x_i, y_j) = \frac{2\pi}{N} \sum_{k=1}^{k=N} A^f(x_i - R \cos(\frac{2k\pi}{N}), y_j - R \sin(\frac{2k\pi}{N}))$$

The integral and discrete formulas for the tube response based on the elementary ‘straw’ element response.

$$A_\text{chan}^f(x, y) = \sum_{i=1}^{i=4} \left[ \int_{\phi=0}^{\phi=2\pi} A^f(x - x_i^{\text{TUBE}} - R \cos \phi, y - y_i^{\text{TUBE}} - R \sin \phi) d\phi \right]$$

One can try to recover the ‘straw’ response map directly from the channel response. All you need - just solve the equation above.
The least squares sum minimization was used to find the ‘straw’ response maps...

\[
S = \sum_{i=1}^{i=M} \sum_{j=1}^{j=M} \left[ A_{\text{tube}}(x_i, y_j) - \frac{2\pi}{N} \sum_{k=1}^{k=N} A_f \left( x_i - R\cos\left(\frac{2k\pi}{N}\right), y_j - R\sin\left(\frac{2k\pi}{N}\right) \right) \right]^2
\]

Which are nothing but profiles of the showers in the calorimeter!

0.6° (upper two plots) and -3.6° (lower two plots).
Just another view of the shower profiles...

Shower profiles and projections (for -0.6° and -5.6° impact angles, 193 GeV electron beam)
Once we have a single channel response map, we can use it for the local response correction

Effect of the position correction on the response spectrum. 193 GeV electron beam, -0.6° (left) and -3.6° (right) impact angle.
Why the stochastic term for the position corrected response is larger?

Once we apply the local position correction, we introduce additional error due to non-perfect position resolution:

\[
100\% \times \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 \Delta^2 x + \left(\frac{\partial F}{\partial y}\right)^2 \Delta^2 y}
\]

Here function \(F\) is a local non-uniformity function with average \(\langle F \rangle = 1\), \(\Delta x\) and \(\Delta y\) are corresponding position resolutions (RMS).

At the same time the local correction improves the resolution by

\[
100\% \times \sqrt{\langle F^2 \rangle - 1}
\]

Once the \(F\) does not change strongly with the energy, the worse low energy position resolution contributes to the stochastic term of the corrected energy resolution.
The Constant (left), Stochastic (center) and Noise (right) terms of the energy resolution as a function of the cluster radius.

193 GeV electrons, -3.6° impact angle, absorber 1.1X₀. Effect of the local position correction (squares) is also presented.
The local response correction allows to compare responses in different channels

Sample set of the Prototype-95 response spectra accumulated for channels occupied with the beam hits. Local position correction applied.
Channel-to-channel response variations
(local position correction applied)

The channel-to-channel variation of the 193 GeV - 3.6° electron response.
Local correction map is shown on the inset.
ATLAS FCal covers angles from $1.2^0$ to $6.0^0$. How uniform the response is in this range of angles?

Angular uniformity of the Prototype response. The worst point is 193 GeV, $4.4^\circ$ angle: 0.7% below the average.