What really goes on in a hadron calorimeter?

Don Groom
deg@lbl.gov

Ernest Orlando Lawrence Berkeley National Laboratory

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What this talk isn’t—

- About any particular kind of calorimeter, except that it be finely sampled
- About any particular simulations, although every package I know about has been used at some stage

In fact, we deal here with an idealized calorimeter which nobody would actually build except for a test beam. It’s the same front to back, and the only place energy can get lost is by front-surface albedo:

Scale of segmentation (absorber and sensor material) small compared to radiation length $X_0$ and hadronic collision length $\lambda$

So pick your favorite absorber—lead, copper, wood, or none. And pick your favorite sensor—Liquid argon, scintillator tiles, quartz fibers, or microphones.
What this talk is—

- An outgrowth of puzzlement about the energy dependence of radiation damage and activation at the SSC
- A discussion of the basic physics processes which lead to the energy-independent quantities $e$ and $h$, and the energy-dependent function $e/\pi$

  $\implies$ Corollary: the $p/\pi$ ratio

- Quantative results concerning resolution

The conceptual framework was laid in a no-longer-so recent paper:

Gabriel, Groom, Job, Mokhov, & Stevenson, CALOR (Geant, CALOR MARS FLUKA ISAJET)


My microscopic view of the detail in a calorimeter owes much to—

  NIM A265, 273 (1988)

- Robert Klaner and associates:
  G. Drews et al., NIM A290, 335 (1990)
What there's no time for—

- Jet response (which follows easily)
- Front-face albedo losses of energy
- Extension to nonuniform calorimeters (separate EM compartment and perhaps hadronic catcher)
  \[\Rightarrow\] Resolution in this case, including correlations
  \[\Rightarrow\] Use of EM/had sections correlations to correct compensation and reduce “constant term”
- Very much detail about anything!
CONCEPTUAL BASIS

- One-way transfer of energy to the EM sector
- The “universal spectrum” of hadronic activity
- e and h

\[ \langle E_e \rangle = F_{\pi^0} E \]
\[ E \quad \langle E_e^{\text{vis}} \rangle = e \langle E_e \rangle \]
\[ \pi^0 \text{ production} \]
\[ \langle E_h \rangle = (1 - F_{\pi^0}) E \]
\[ h \quad \langle E_h^{\text{vis}} \rangle = h \langle E_h \rangle \]

\[ \langle E^{\text{vis}} \rangle = \langle E_e^{\text{vis}} \rangle + \langle E_h^{\text{vis}} \rangle \]

\[ \langle E \rangle \]

\[ \text{In each high-energy collision in the developing cascade, } \approx 1/3 \text{ of the energy is transferred to the EM sector} \]

Except for a stray photoneutron or two, it doesn’t come back

The mean transfer fraction \( F_{\pi^0} \) increases with energy, and has an asymptote at unity
We'll return to the all-important energy transfer later, but first a word about the "universal spectrum:

Almost all hadronic energy is deposited by very low-energy particles. The spectrum and composition are independent of energy or species of the original hadron.
The heart of all compensation and most resolution issues is in the transfer to the EM sector

\[ \langle E_e \rangle = F_{\pi^0} E \]

\[ \langle E^\text{vis}_e \rangle = e \langle E_e \rangle \]

\[ \langle E^\text{vis}_h \rangle = \langle E^\text{vis}_e \rangle + \langle E^\text{vis}_h \rangle \]

\[ \langle E^\text{vis}_h \rangle = h \langle E_h \rangle \]

\[ \langle E_h \rangle = (1 - F_{\pi^0}) E \]

\[ \langle E^\text{vis}_h \rangle = h \langle E_h \rangle \]

Visible signal = \( eE_e + hE_h \)
$f_0(E_e)$, the distribution of EM energy in a hadron-induced cascade

![Graph showing the distribution of EM energy in a hadron-induced cascade]

- 20 GeV
- 200 GeV (scale × 2)

![Graph showing the fractional mean and variance]

$1 - (E/E_0)^{m-1}$

$E_0 = 1$ GeV, $m = 0.85$

$E_0 = 0.764$ GeV, $m = 0.866$

$\sigma_{nc}(E) = 0.171 - 0.010 \ln(E/(1 \text{ GeV}))$

$\sigma_0/E$
... and there is a very, very interesting further step

Each generation

- degrades the energy by a factor equal to the multiplicity $n$, which is nearly constant
- and moves $f_{\pi^0} \approx 1/3$ of the energy out of the hadronic sector

This induction rule amounts to a power law:

$$E_h \propto E^m$$

$$= E_0 (E/E_0)^{m-1}$$

$$F_h = (1 - F_{\pi^0}) = (E/E_0)^{m-1}$$

where

$$m = 1 - \frac{\ln(1/(1 - f_{\pi^0})}{\ln n} \approx 0.83$$

On physics grounds, $E_0$ ought to be $\approx 1$ GeV; fits to simulation results give values in this range or a little lower.
If you don’t believe a derivation, maybe you will believe the many, many simulations, *e.g.*

\[
\frac{(E/0.96)^{0.816-1}}{\chi^2 = 16.5}
\]

\[
\frac{(E/2.62)^{0.814-1}}{\chi^2 = 5.9}
\]

...and there’s even more physics in this. In a hadronic collision, a leading hadron *like the incident hadron except maybe for charge* carries the lion’s share of the energy. A leading pion has a good chance of being neutral, so that a large energy fraction moves to the EM sector.

- On average, therefore, pions have a smaller \( F_h \) than protons, and in a noncompensating calorimeter give a different response.

Wigmans *et al.* have measured this effect in a quartz-fiber calorimeter.
The bottom line of all this has to be the response of a hadron calorimeter, so we return to the main flow diagram:

\[
\begin{align*}
\langle E_e \rangle &= F_{\pi^0} E \\
\langle E_e^{\text{vis}} \rangle &= e \langle E_e \rangle \\
\langle E_{h} \rangle &= (1 - F_{\pi^0}) E \\
\langle E_{h}^{\text{vis}} \rangle &= h \langle E_{h} \rangle
\end{align*}
\]

\[
\langle E^{\text{vis}} \rangle = \langle E_e^{\text{vis}} \rangle + \langle E_{h}^{\text{vis}} \rangle
\]

\[f_0(E_e) \quad \pi^0 \text{ production} \quad f_e(E_e^{\text{vis}} | E_e) \quad f_h(E_{h}^{\text{vis}} | E_e)
\]

\[
e \text{ and } h \text{ are in themselves interesting quantities, absolutely definable and measureable. But for lack of time, we just define them:}
\]

\[e \text{ is the fraction of EM energy resulting in a visible signal, usually measured in MIPS (in itself a troublesome quantity)}
\]

\[h \text{ is the similar fraction for hadronic energy. It is usually smaller because of "invisible" energy deposit}
\]

So the total signal, e.g. for an incident proton, is

\[E_{h}^{\text{vis}} = e F_{\pi^0} E + h F_{h} E
\]

\[= [1 - (1 - h/e) F_{h}] e E
\]

Since \(E_e^{\text{vis}} = eE\), we immediately have

\[E_e^{\text{vis}} / E_{h}^{\text{vis}} (= e/\pi) = [1 - (1 - h/e) F_{h}]^{-1}
\]
With our powerlaw form for $F_h$, we can immediately take this a couple of steps further:

$$E_e^{vis} / E_h^{vis} (= e / \pi) = \left[1 - (1 - h/e)F_h\right]^{-1}$$

$$= \left[1 - (1 - h/e)(E/E_0)^{m-1}\right]^{-1}$$

$$= \frac{1}{1 - aE^{m-1}}$$

One can do fits to measurements to find $a$ and $m$; unfortunately, one must depend on theory or other input to extricate $e/h$ and $E_0$. 
We have tested these functional forms on virtually all data which has come our way. First, some very old test beam data:

(See better example by Jianke Liu, this conference.)
The results are ugly but contain new insights:

\[
\left(\frac{\sigma}{E}\right)^2 = \frac{(1 - F_{\pi^0})(\sigma_{h0}^2 h/e - \sigma_{e0}^2)}{E} + (1 - h/e)^2 \sigma_{nc}^2 + \sigma_{t}^2 / E \quad \sigma_{t}^2 \frac{E_{vis}}{E^2}
\]

Since for a real calorimeter

\[\sigma_{h0}^2 >> \sigma_{e0}^2\]

we can drop the \(\sigma_{e0}^2\) terms, and obtain

\[
\left(\frac{\sigma}{E}\right)^2 \approx \frac{(1 - F_{\pi^0})\sigma_{h0}^2 h/e}{E} + (1 - h/e)^2 \sigma_{nc}^2 + \sigma_{t}^2 \frac{E_{vis}}{E^2}
\]

\[\approx \frac{(E/E_0)^m \sigma_{h0}^2 h/e}{E} + (1 - h/e)^2 \sigma_{nc}^2 + \sigma_{t}^2 \frac{E_{vis}}{E^2}
\]

\[\approx \left(\frac{\sigma'}{E^{0.57}}\right)^2 + (1 - h/e)^2 \sigma_{nc}^2 + \sigma_{t}^2 \frac{E_{vis}}{E^2}
\]

The Wigmans group quartz-fiber calorimeter is special, in that the transducer term is always more important than the fluctuation-produced "1/E" term. Moreover, it "sees" mostly the EM sector. With a redefined \(\sigma_t\), we then have

\[
\left(\frac{\sigma}{E}\right)^2 \approx \frac{\sigma_t^2 (1 - aE^{m-1})}{E} + (1 - h/e)^2 \sigma_{nc}^2
\]

**Prediction:** Fit \(a\) to his \(\pi/e\), then plot \((\sigma/E)^2\) as a function of \((1 - aE^{m-1})/E\)

\[\rightarrow You\ should\ get\ a\ straight\ line\]
Conclusions

This approach seems to be robust, and provides the correct asymptotes and (so far) a correct description of available data.

Some of the input parameters, e.g., absolute values of $e$ and $h$, and better descriptions of the internal statistical distributions, can be determined by future test beam work e.g. {\textit{Wigman's "dual readout" calorimeter}}.

Extension to more real calorimeters, e.g. ones with separate EM compartments, have yet to be completed, but these promise virtual elimination of the constant term by use of EM/had compartment correlations.